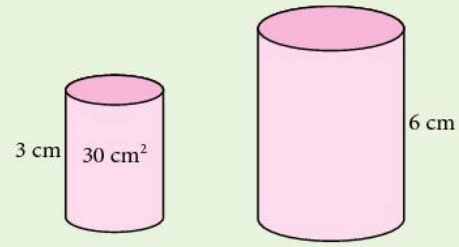
Example 1

Two similar cylinders have heights of 3 cm and 6 cm respectively. If the volume of the smaller cylinder is 30 cm³, find the volume of the larger cylinder.



If linear scale factor = k, then ratio of heights $(k) = \frac{6}{3} = 2$

$\therefore$ ratio of volume $(k^3) = 2^3$

$= 8$

and volume of larger cylinder $= 8 \times 30$

$= 240 \text{ cm}^3$

Example 2

Two similar spheres made of the same material have masses of 32 kg and 108 kg respectively. If the radius of the larger sphere is 9 cm, find the radius of the smaller sphere.

We may take the ratio of masses to be the same as the ratio of volumes.

ratio of volume $(k^3) = \frac{32}{108}$

$= \frac{8}{27}$

ratio of corresponding lengths $(k) = \sqrt[3]{\left(\frac{8}{27}\right)}$

$= \frac{2}{3}$

$\therefore$ Radius of smaller sphere $= \frac{2}{3} \times 9$

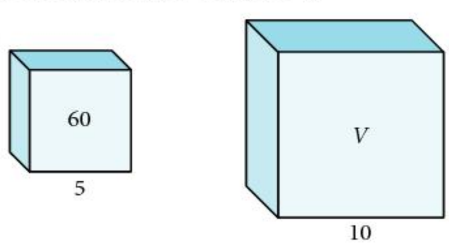
$= 6 \text{ cm}$

Exercise 9

In this exercise, the objects are similar and a number written inside a figure represents the volume of the object in cm³.

Numbers on the outside give linear dimensions in cm. In questions **1** to **8**, find the unknown volume V.

1.



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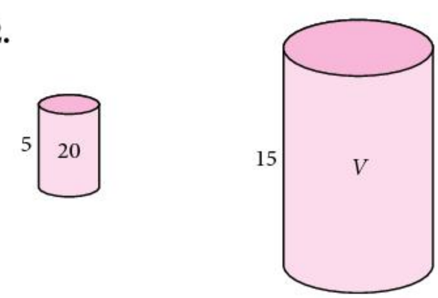

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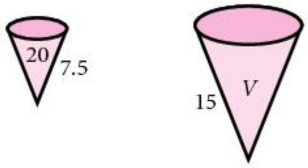

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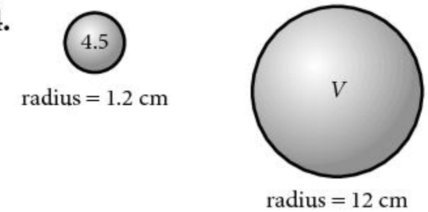

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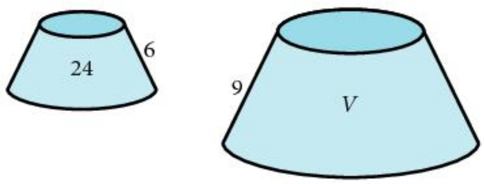

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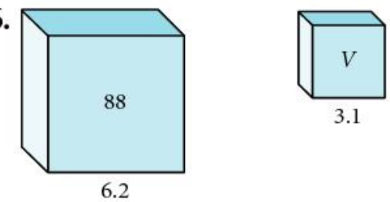

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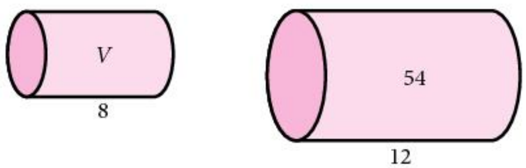

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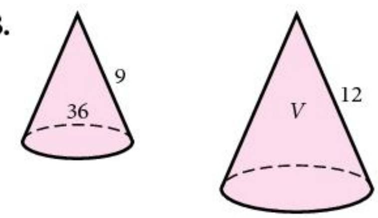
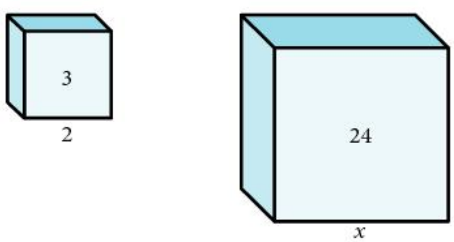


In questions **9** to **14**, find the lengths marked by a letter.

9.



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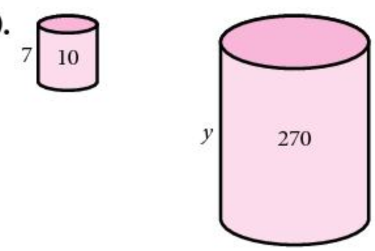

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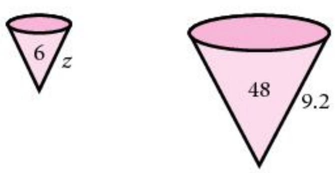

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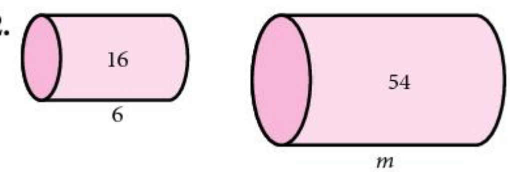

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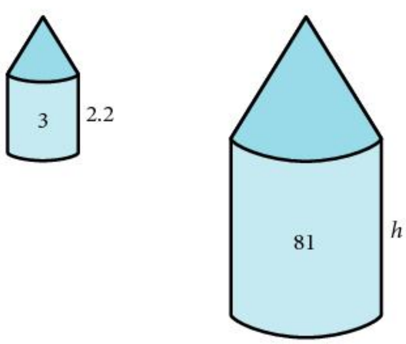

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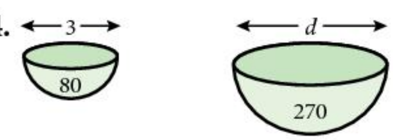


15. Two similar jugs have heights of 4 cm and 6 cm respectively. If the capacity of the smaller jug is 50 cm^3, find the capacity of the larger jug.

16. Two similar cylindrical tins have base radii of 6 cm and 8 cm respectively. If the capacity of the larger tin is 252 cm^3, find the capacity of the smaller tin.

17. Two solid metal spheres have masses of 5 kg and 135 kg respectively. If the radius of the smaller one is 4 cm, find the radius of the larger one.

18. Two similar cones have surface areas in the ratio 4 : 9. Find the ratio of:

a) their lengths **b)** their volumes.

19. The area of the bases of two similar glasses are in the ratio 4 : 25. Find the ratio of their volumes.

20. Two similar solids have volumes V_1 and V_2 and corresponding sides of length x_1 and x_2. State the ratio $V_1 : V_2$ in terms of x_1 and x_2.

21. Two solid spheres have surface areas of 5 cm^2 and 45 cm^2 respectively and the mass of the smaller sphere is 2 kg. Find the mass of the larger sphere.

22. The masses of two similar objects are 24 kg and 81 kg respectively. If the surface area of the larger object is 540 cm^2, find the surface area of the smaller object.

23. A cylindrical can has a circumference of 40 cm and a capacity of 4.8 litres. Find the capacity of a similar cylinder of circumference 50 cm.

24. A container has a surface area of 5000 cm^2 and a capacity of 12.8 litres. Find the surface area of a similar container which has a capacity of 5.4 litres.

4.5 Congruence

Two plane figures are congruent if one fits exactly on the other.
They must be the same shape and size.

In order to prove that two triangles are congruent, you must prove *one* of the following criteria:

SSS: that all three pairs of corresponding sides are equal.

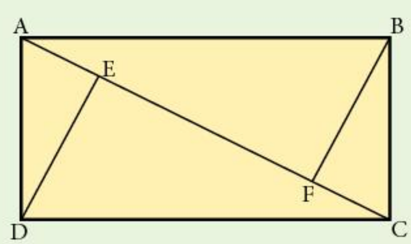
ASA: that two pairs of corresponding angles are equal, along with one pair of corresponding sides.

SAS: that two pairs of corresponding sides are equal, along with the corresponding *included* angles.

RHS: In the special case of right-angled triangles, you can prove congruence by showing that the hypotenuses are equal in length, along with one pair of corresponding sides.

Example

ABCD is a rectangle. Points E and F are positioned along the diagonal such that AC is perpendicular to both DE and BF.



Prove that triangles DEC and BFA are congruent.

$D\hat{E}C = B\hat{F}A$ (both right-angles)

$DC = AB$ (opposite sides of a rectangle)

$D\hat{C}E = B\hat{A}C$ (alternate angles)

Hence by the criteria **RHS**, it has been proved that triangles DEC and BFA are congruent.

Make sure you give reasons for all of the statements that you make and give a conclusion at the end.