🤑: Greedy algorithm.

☢️: Dynamic Programming.

➗: Divide-and-Conquer.

🤑 Ford–Fulkerson Method (Maximum Flow) `O(E |f*|)`

Inputs: Given a Network `G=(V,E)` with flow capacity `c`, a source node `s`, and a sink node `t`.
Output: maximum flow `f` from `s` to `t`.

```
for all edges (u,v):
    f[u, v] := 0
while there is a path p from s to t in Gf, such that cf(u,v) > 0 for all edges
(u,v) in p:
    cf(p) := min([cf(u, v) for each edge (u, v) in p])
    for each edge (u,v) in p:
        f(u, v) += cf(p) # Send flow along the path
        f(v, u) -= cf(p) # The flow might be "returned" later
```

🤑 Edmonds–Karp algorithm (Implementation of FFM) `O(V·E²)`

An implementation of the Ford–Fulkerson method.

```
for all edges (u,v):
    f[u, v] := 0
while, according to BFS, there is a path p from s to t in Gf (assuming unitary
distance on every edge):
    cf(p) := min([cf(u, v) for each edge (u, v) in p])
    for each edge (u,v) in p:
        f(u, v) += cf(p) # Send flow along the path
        f(v, u) -= cf(p) # The flow might be "returned" later
```

🤑 Ford–Fulkerson Method (Maximum Bipartite Matching) `O(E |f*|)`

Input: a bipartite graph $G = (V, E)$ with $V = L \cup R$.

Output: Size of maximum matching.

1. Build the flow network:
    1. For every (u,v)∊E, assign capacity c(u, v) = 1.
    2. Add source node s and sink node t.
    3. For every u ∊ L, add edge (s, u) with capacity c(s, u) = 1 .
    4. For every v ∊ R, add edge (v, t) with capacity c(v, t) = 1 .
2. Apply `Ford–Fulkerson`. Return the output value.

🤑 Ford–Fulkerson (Approx. Minimum Bipartite Vertex Cover) `O(E |f*|)`

Input: an undirected graph $G = (V, E)$.

Output: 2-approximation to the minimum size of vertex cover in G.

Just use 🤑 Ford–Fulkerson Method (Maximum Bipartite Matching) `O(E |f*|)`.

This is because the Maximum Bipartite Matching is a 2-approximation to the Min. Bipartite Vertex Cover.

🤑 Approximate Minimum Vertex Cover `O(V+E)`

Input: an undirected graph $G = (V, E)$.

Output: 2-approximation to the minimum size of vertex cover in G.

```
C = []
E' = G.E
while E' is not []:
    Randomly select edge (u, v) from E'
    C.append((u, v))
    remove every edge connecting u or v in E'
return C
```

🤑 Exact Subset-Sum `O(exp)`

```
def exact_subset_sum (S, t):
    n = len(S)
    L[0] = {0}
    for i in range(n):
        L[i] = sorted( unique( L[i-1] + L[i-1] + {S[i]} ) )
        L[i] = filter(lambda x: x<=t, l[i])
    return max([sum(l) for l in L])
```

🤑 Approx. Subset-Sum `O(poly)`

```
def approx_subset_sum (S, t, e):
    def trim(l, d):
        '''removes elements within `d` of its predecessor.'''
        m = len(l)
        l` = {l[0]}
        last = l[0]
        for i in range(2, m):
```

```
            if l[i] > list*(1+d): # because l is sorted
                l`.append(l[i])
                last = l[i]
        return l` # a trimmed, sorted list
    n = len(S)
    L[0] = {0}
    for i in range(n):
        L[i] = sorted( unique( L[i-1] + L[i-1] + {S[i]} ) )
        L[i] = trim(L[i], e/2/n)
        L[i] = filter(lambda x: x<=t, l[i])
    return max([sum(l) for l in L])
```

[GraphTrav] BFS: Breadth-First Search `O(V+E)`

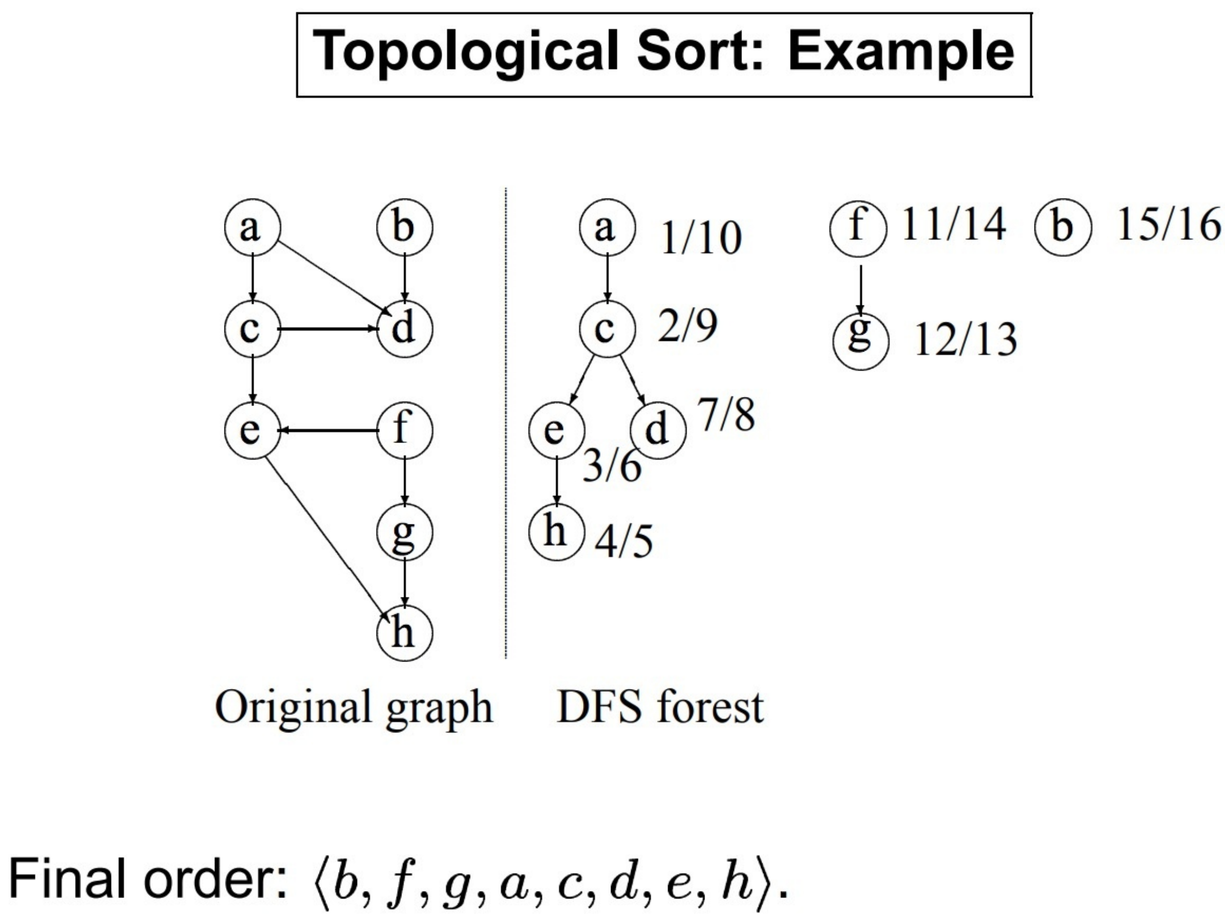
```
def BFS(G, s):
    # Mark all the vertices as not visited
    visited = [False]*(len(G.V))
    # Create a queue for BFS, enqueue s:
    queue = [s]
    # Mark the source node as visited:
    visited[s] = True
    while queue:
        # Dequeue a vertex from queue and print it
        s = queue.pop()
        print s,
        # Get all adjacent vertices of the dequeued
        # vertex s. If a adjacent has not been visited,
        # then mark it visited and enqueue it
        for i in G.neighbors[s] if not visited[i]:
            queue.append(i)
```

[GraphTrav] DFS: Depth-First Search `O(V+E)`

```
def DFSUtil(G,v,visited):
    '''A function used by DFS'''
    visited[v] = True # Mark the current node as visited
    print v, # print the current node
    # Recur for all the vertices adjacent to this vertex
```

```
    for i in G.neighbors[v]:
        if visited[i] == False:
            G.DFSUtil(i, visited)

def DFS(G,v):
    '''The function to do DFS traversal. It uses recursive DFSUtil()'''
    visited = [False]*(len(G.V)) # Mark all the vertices as unvisited
    for i in range(V):
        if visited[i] == False:
            G.DFSUtil(v,visited) # Call the recursive helper function to print DFS traversal
```

[GraphTrav][ShortestPath] Topological Sort (DAG Only; Allows w<0; Single-Source) `O(V+E)`



1. Run DFS(G), computing finish time for each vertex;
2. As each vertex is finished, insert it onto the front of a list;

3. Output the list.

```
def topologicalSortUtil(G, v, visited, stack):
    '''A recursive function used by topologicalSort'''
    visited[v] = True # Mark the current node as visited.
    # Recur for all the vertices adjacent to this vertex
    for i in G.neighbors[v]:
        if not visited[i]:
            G.topologicalSortUtil(i, visited, stack)
    stack.insert(0,v) # Push current vertex to stack which stores result

def topologicalSort(G):
    '''The function to do Topological Sort.
       It uses recursive topologicalSortUtil()'''
    visited = [False]*G.V # Mark all the vertices as not visited
    stack   = []
    # Call the recursive helper function to store Topological
    # Sort starting from all vertices one by one
    for i in range(G.V):
        if not visited[i]:
            G.topologicalSortUtil(i, visited, stack)
    print stack # Print contents of stack
```

🤑[ShortestPath] Dijkstra (Allows Cycles; No weight<0; Single-Source) `O(V²)→O(V·logV)`

```
def initialize_single_source(graph, source):
    for each vertex v in graph:
        v.d = ∞
        v.π = None
    s.d = 0

def relax(u, v, weight_of_edge_uv):
    if v.d > u.d + weight_of_edge_uv:
        v.d = u.d + weight_of_edge_uv
        v.π = u

def extract_min(set_of_vertices):
    a = vertex in set_of_vertices whose distance d is min
    set_of_vertices.pop(a)
      return a
```

```
def dijkstra(G, w, s):
    initialize_single_source(G, s)
    S = []
    Q = G.Vertices
    while Q is not empty:
        u = extract_min(Q)
        S.append(u)
        for each vertex v in G.adj[u]:
            relax(u, v, w[u, v])
```

☢️[ShortestPath] Bellman-Ford (Allows Cycles; Allows weight<0; Single-Source) `O(V·E)`

```
procedure BellmanFord(list vertices, list edges, vertex source)
   // 该实现读入边和节点的列表，并向两个数组（distance和predecessor）中写入最短路径信息

   // 步骤1：初始化图
   for each vertex v in vertices:
       if v is source then distance[v] := 0
       else distance[v] := infinity
       predecessor[v] := null

   // 步骤2：重复对每一条边进行松弛操作
   for i from 1 to size(vertices)-1: // repeat n-1 times -- iteration ID not important:
       for each edge (u, v) with weight w in edges:
           if distance[u] + w < distance[v]: // if taking this edge yields shorter dist.:
               distance[v] := distance[u] + w // relax dist. to v via this route:
               predecessor[v] := u // record the current best solution.

   // 步骤3：检查负权环
   for each edge (u, v) with weight w in edges:
       if distance[u] + w < distance[v]:
           raise "图包含了负权环"
```

☢️ Bellman-Ford (Negative Cycle Detection) `O(V·E)`

> 1. Color every node white.
> 2. For each node u (in an arbitrary order),
>   1. set v := u;
>   2. while v is white and has a predecessor,
>   1. recolor v gray;
>   2. set v := predecessor[v].
>   3. If v is gray, we found a cycle:
>
>   loop through again to read it off.
>
>   Else, none of the gray nodes are involved in a cycle;
>
>   loop through again to recolor them black.
>
> Source: [algorithms - Finding the path of a negative weight cycle using Bellman-Ford - Computer Science Stack Exchange]()

☢️[ShortestPath] Matrix Multiplication (All-Pair) `θ(n³ lg n)`

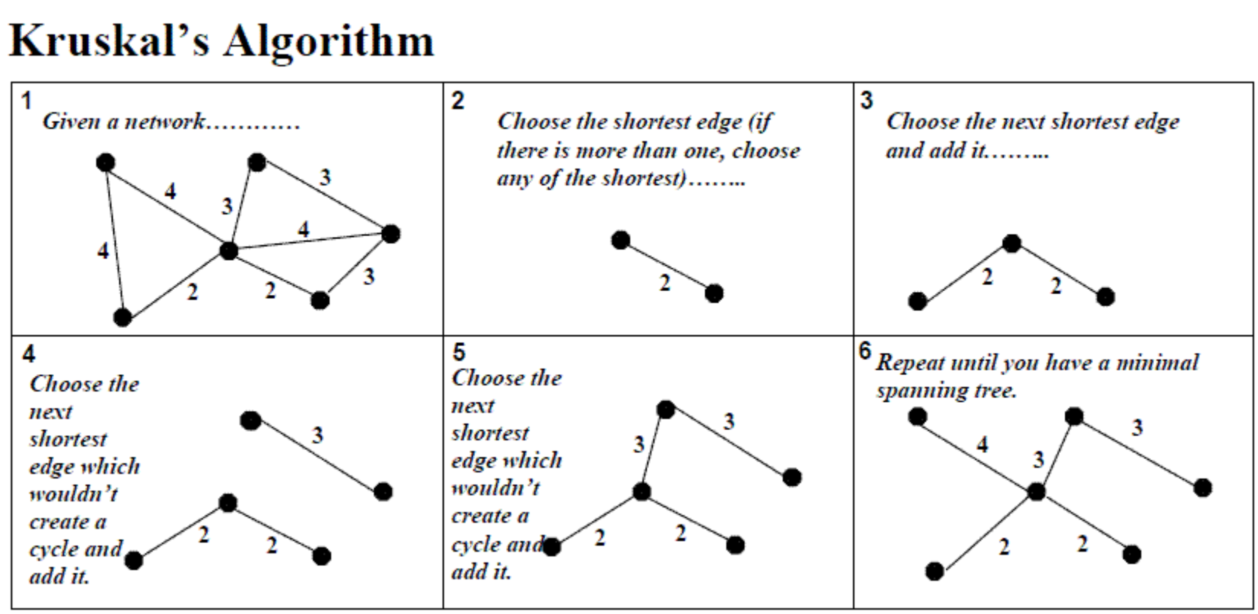
```
def extend_shortest_paths(L, W):
    n = L.rows
    Let M be a new n*n matrix
    for i in range(n):
        for j in range(n):
            M[i, j] = ∞
            for k in range(n):
                M[i, j] = min(M[i, j], M[i, k] + W[k, j])
                # If by taking route k i can reach j faster, then take this path.
                # Otherwise, remain the shortest path length unchanged.
    return M

def faster_all_pairs_shortest_paths(W):
    n = W.rows # get size of square matrix W
    L = {1: W}
    m = 1
    while m < n-1:
        L[2*m] = extend_shortest_paths(L[m], L[m])
```

```
        m *= 2 # we have 1, 2, 4, 8, ..., n-1
    return L[m]
```

☢️[ShortestPath] Floyd-Warshall (All-Pair) `Θ(n³)`

```
let dist be a |V| × |V| array of minimum distances initialized to ∞
for each vertex v:
   dist[v][v] ← 0
for each edge (u,v):
   dist[u][v] ← w(u,v)  // the weight of the edge (u,v)
for k from 1 to |V|:
   for i from 1 to |V|:
      for j from 1 to |V|:
         if dist[i][j] > dist[i][k] + dist[k][j] :
            dist[i][j] ← dist[i][k] + dist[k][j]
```

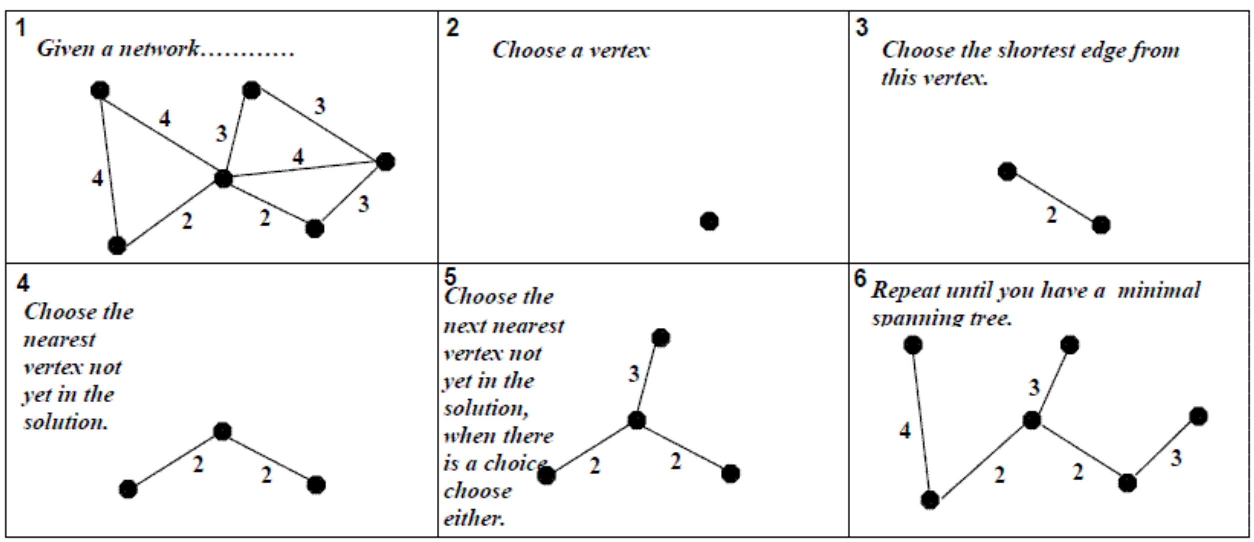
🤑[MST] Kruskal's Algorithm (take shortest; for undirected) `O(E·logV)`



```
A = {}
for v in G.V:
   v = set(v)
for (u, v) in G.E increasingly ordered by weight(u, v):
   if FIND-SET(u) ≠ FIND-SET(v): # if adding this edge won't incur cycles:
      A.append( (u, v) )
      UNION(u, v)
return A
```

🤑[MST] Prim's (take nearest; for undirected & connected) `O(E·logV)→O(E+V·logV)`

Prim's Algorithm




```
T = {}
U = { random.choice(V) }
while U ≠ V: # Before U includes all vertices in G, repeat:
    Find the "light edge" (u, v) s.t. u ∈ U and v ∈ V - U # Find the nearest
vertex to (and thus not yet in) U:
    T.append( (u, v) )
    U.append( v )
```

🤑 Recursive Activity Selection

```
s = { array of starting times }
f = { array of finishing times } # we assume that activities are ordered by
monotonically increasing finish time
n = number of activities
def recursively_select_activity(k):
    m = k+1 # Start search from the next planned activity.
    while m<=n and s[m]<f[k]: # As long as m is not the last planned activity
and that m wants to start before k ends:
        m += 1 # Go on searching.
    if m<=n: # if finally found such one:
        return {a_m}\cup recursively_select_activity(m)
    else: # if not:
        return {}

recursively_select_activity(0)
```

🤑 Iterative Activity Selection

```
# Input:
s = { array of starting times }
f = { array of finishing times } # we assume that activities are ordered by
monotonically increasing finish time
n = number of activities
# Init:
A = {a_1}
k = 1
# Main loop:
for m = 2 to n:
    if s[m]>=f[k]:
        A.append(a_m)
        k = m
return A
```

☢️ 0-1 Knapsack Problem

```
def knapSack(W, wt, val, n):
    '''
    # A Dynamic Programming based Python Program for 0-1 Knapsack problem
    # Returns the maximum value that can be put in a knapsack of capacity W
    W = total weight carry-able
    wt = { array of items' weights }
```

```
    val = { array of items' values }
    n = total number of items
    '''
    K = {{ (n+1)-by-(W+1) matrix of 0 }}
    # Build table K[][] in bottom up manner
    for i in range(n+1): # When taking the first i items:
        for w in range(W+1): # When there is w capacity left:
            if i==0 or w==0: # if it's "nothing" or that this slot is empty:
                K[i][w] = 0 # Max value we can get from this situation is 0.
            elif wt[i-1] <= w: # else, if the remaining capacity can accomodate the item:
                K[i][w] = max(val[i-1] + K[i-1][w-wt[i-1]],  K[i-1][w]) # set the value at this slot to be the max one of the two options: (1) add this item, shrinking the remaining capacity by its weight; (2) pass this item, leaving the remaining capacity unoccupied.
            else: # there's no space to accomodate this item:
                K[i][w] = K[i-1][w] # we can only pass this item.
    return K[n][W]
```

🤑 Fractional Knapsack Problem

```
Sort list of items by value-to-weight ratio.
While knapsack is not full and list of items is not exhausted:
    A = first item in the list.
    Put as much A as possible into the knapsack.
```

🤑 Huffman (Optimal Prefix Coding) `O(n·lgn)→O(nlglgn)`

```
n = len(C)
Q = C
for i in range(n-1):
    x = Q.pop_min()
    y = Q.pop_min()
    z = new Node(
        left = x,
        right = y,
        freq = x.freq + y.freq)
    Q.append(z)
assert len(Q) == 1 and Q[0].freq == 1.0
return Q[0]
```

🤑 Maximum-Weight Indep. Subset of A Matroid

Given a matroid $M = \{S, I\}$ and its associated weight vector w.

```
A = []
Sort M.S by monotonically decreasing weight w.
for x in M.S:
    if A+{x} is still independent: # i.e. A+{x} is in M.I:
        A.append(x)
return A
```

➗ Linear Select (Select the k-th-big item with linear time even in worst case) `O(n)`

```
def select(a, i):
    if len(a)<5: return sorted(a)[i]
    #else:
    a_rect = reshape_to_5(a) # 5 items per group (row).
    m = [ median(row) for row in a_rect ]
    if len(m) % 2 == 0: # if even items
        median_to_get = (len(m)-1)/2
    else: # odd items:
        median_to_get = len(m)/2
    x = select(m, i = median_to_get) # use SELECT to find the median-of-
medians.
    # partition:
```

```
    l = a[ np.where( a < x ) ] # lower half
    h = a[ np.where( a > x ) ] # higher half
    # locate desired value:
    k=len(l)
    if  i==k:
        return x
    elif i<k:
        return select(l, i)
    elif i>k:
        return select(h, i-k-1)

result = select(a,i)
assert result==sorted(a)[i]
```

➗ Quick Select (T(n) = T(n/2) + n) `O(n)`

```
def select(a, k):
    n = len(a)
    if n==1: return a[0]
    #else:
    pivot = random.choice(a)
    # construct a result array:
    l = []
    e = []
    h = []
    # group every item according to comparasion to the pivot:
    for this in a:
        if this<pivot:   l.append(this)
        elif this>pivot: h.append(this)
        else:            e.append(this)
    if len(l)+len(e)<=k:
        k -= len(l) + len(e)
        a = h # find in the higher group
    elif len(l)<=k:
        k -= len(l)
        a = e # find in the "equal" group
    else: #k<len(l)
        a = l # find in the lower group
    if len(h)==0 and len(l)==0:
        return pivot # A-hah! The pivot happens to be just the target value!
    else: # ge ming shang wei cheng gong, tong zhi men reng xu nu li:
        return select(a, k)
```

➗ Quick Sort (T(n) = 2T(n/2) + n) `O(n log n)`

```
def sort(a):
    n = len(a)
    if n<=1: return a
    #else:
    pivot_id = np.random.choice(n)
    pivot    = a[pivot_id]
    # construct a result array:
    l = []
    h = []
    for i in range(n):
        this = a[i]
        if i!=pivot_id: # Be aware that we use the ID to allow for non-pivot items with the same value as the pivot.
            if this<pivot:
                l.append(this)
            else:
                h.append(this)
    return sort(l)+[pivot]+sort(h)
```

➗ Interleaves Two Halves of An Array (T(n) = 2T(n/2) + n/4) `θ(n log n)`

```
def interleave(start, end):
    n     = (end-start)/2
    mid   = n/2
    cycle = n-mid
    for i in range(start+mid,start+n):
        swap(a[i], a[i+cycle])
    if n > 2:
        interleave(start, start+n)
        interleave(start+n, start+2*n)
```